

Fig. 3 Peak intensity ratio of second pulse to initial pulse for collimated beams,  $N_P = 2$ .

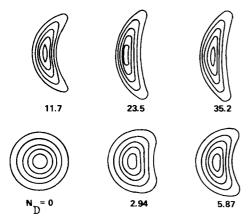


Fig. 4 Constant intensity contours for collimated beams,  $N_P = 2$ .

## **Conclusions**

Presented here are results that demonstrate the improvement in peak intensity obtained by propagation of lowpulse-rate beams compared to CW beams of the same average transmitted power. These results have been obtained without consideration for such practical limitations as atmospheric breakdown, which could be encountered with low-pulse-rate, high-average-power pulsed lasers. These results are subject to the stated conditions of negligible pressure relaxation, negligible single pulse blooming effects, and the assumed Gaussian beam profile. However, it should be pointed out that the method of solution presented is not limted to these restrictive conditions, but may be employed for analyzing lasers with non-Gaussian beam profiles, beams from oscillators with central obscuration, or nonsymmetric beam profiles. The basic method of solution used here has now been extended by other investigators to analyze the effects of atmospheric turbulence, transonic slewing beams, beams with stagnation zones, and beams from lasers with compensating optical output to further reduce thermal-blooming effects.

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# Total Head/Static Measurements of Skin Friction and Surface Pressure

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#### Introduction

EASUREMENT of the local friction force is an important objective in many experiments. Yet it seems that no method is both accurate and practical in all applications of potential interest. Various methods have been used, among which the Preston tube<sup>2,3</sup> seems to be one of the most simple and reliable. By measuring the differential between the total head and the local static pressure it is possible to obtain the friction force from a calibration curve. But it is often difficult or costly to measure the static pressure with the required accuracy. This may be the case for instance near the trailing edge of a wing model, and generally on models not originally intended for static pressure investigations. It is also difficult to establish where a static pressure tap should be located to be undisturbed by the presence of the Preston tube. Different remedies to this problem have been suggested; among others Gupta<sup>4</sup> suggested the use of a double Preston tube of different outer diameters and with one tube cut at 45°.

In the present report, another approach is suggested. In view of all the difficulties encountered in obtaining the correct static pressure and the necessity of drilling holes in the surface (or placing a static pressure probe somewhere outside the boundary layer) it appears simpler to measure the static pressure in the center plane of the tube at a specified downstream distance from the leading edge of the tube provided this can be related to the actual static pressure at the tip. This will not be the true static pressure, but the deviations are small and can be accounted for. The probe may be located anywhere within the measurement area like the Gupta probe but will give larger differential pressures than the latter, which is desirable for improved accuracy. One disadvantage of the present method is that small probes are difficult to make.

The use of the new probe is based on knowledge of the flow around surface cylinders aligned with the flow in a nominally constant pressure boundary layer. In Ref. 5 such information has been collected in the form of velocity profiles, pressure distributions, and flow visualizations. It appears that a static pressure tap located at a distance L/D > 2 downstream of the leading edge is not much influenced by the local pressure gradient due to the flow field around the cylinder. The pressure tap should preferably be located outside the pressure peak in the vicinity of the leading edge, but should be as near the front as possible to minimize the influence from external longitudinal pressure gradients. As a compromise the probe illustrated in Fig. 1 was made, with dimensions D=3 mm, L/D = 5. A limited investigation was undertaken to determine the L/D effects, covering L/D=5 to 2. In the arguments previously presented the wall thickness has been ignored as it

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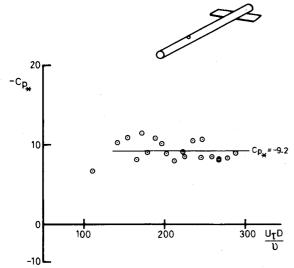


Fig. 1 Static pressure coefficient  $C_p$ , as a function of tube Reynolds number  $U_{\tau}D/\nu$ . Total head/static tube D=3 mm, L/D=5.

is assumed that for t=d/D>0.6 all probes give outputs independent of this ratio.<sup>6</sup>

#### **Analysis**

The usual Preston tube measurement consists of a total pressure  $p_{tot}$  and a static pressure  $p_s$ , both supposed to represent the true values at the probe tip. The present probe also senses the true total pressure at the probe tip, but the static measurement is a displaced static pressure  $p_*$  at the location of the pressure tap (Fig. 1). Thus the pressure difference read by the new probe is

$$\Delta p_* = p_{tot} - p_* = p_{tot} - p_s + p_s - p_* \tag{1}$$

The corresponding Preston tube differential pressure is denoted  $\Delta p$ 

$$\Delta p = p_{tot} - p_s$$

Like the Preston tube the new probe must be located within the wall layer and it is reasonable to assume that the pressure distribution around it scales with the friction velocity  $U_{\tau}$ .

It follows that the static pressure felt by the probe for zero pressure gradient basic flow can be expressed as

$$p_* = f \left( \rho U_\tau^2, \ L/D \right) \tag{2}$$

along the center plane. This functional relationship can be rewritten as, for zero pressure gradient:

$$p*-p_s = C_{p_*} \cdot (\rho/2) U_\tau^2 \text{ where } C_{p^*} = f(L/D)$$
 (3)

 $C_p$ . is the pressure coefficient relative to the true static pressure at the tip. Thus  $\Delta p_\star/\tau = (\Delta p/\tau) - (C_{p^\star}/2)$ . This is a relationship between the calibration parameter  $\Delta p_\star/\tau$  for the new probe and the corresponding  $\Delta p/\tau$  for a Preston tube. It should be noted here that the difference  $C_{p^\star}/2$  between the two is not affected by the choice of Preston tube calibration used, since  $C_{p^\star}$  is mainly a function of L/D.

In earlier experiments a Preston tube calibration was found of form

$$\Delta p/\tau = A \log_{10}(u_{\tau}D/\nu) + B$$
where  $A = 87.77$ ;  $B = 51.93$  (4)

From the equations in the preceding it appears plausible to assume that the new probe has a calibration curve of the form

$$\Delta p_{\star}/\tau = A \log_{10} (U_{\tau}D/\nu) + B - (C_{p^{\star}}/2)$$
 (5)

As  $C_{p^*} = f(L/D)$  the choice of location for the static pressure tap specifies a certain  $C_p$ , value for the probe. Thus the calibration curve for the new probe should be equal to the one for a Preston tube, except for a change in the additive constant equal to  $-C_{p^*}/2$ .

When accounting for the effects of external pressure gradients, changes in both total and static pressures have to be considered. The pressure gradient may affect the velocity profile in the wall region, in which case the calibration and use of both the Preston tube and new probe is in doubt. But the Preston tube is known to work in moderate to small pressure gradients and the same can be assumed to hold for the new probe.

A suggestion is made in the following for the order of magnitude of pressure gradient effects on the total head/static probe, assuming that the total pressure is not affected. The assumption used is that due to the external pressure gradient,  $p_*$  is changed by an amount simply equal to the increase in the external static pressure over the same length.

This yields

$$\frac{\Delta p_*}{\tau} = \frac{\Delta p}{\tau} - \frac{C_{p^*}}{2} + \frac{L}{\rho U_*^2} \cdot \frac{\mathrm{d}p}{\mathrm{d}x} \tag{6}$$

for the case of a static pressure tap a distance L from the leading edge when the calibration is done in a nominally zero pressure gradient flow. (It is assumed that dp/dx is small).

#### **Experiments**

A probe of 3-mm outside diameter and 60 mm length was made from thin-walled brass tubing (Fig. 1) with a ratio of inner to outer diameter of 0.8. A 1-mm tube with closed front end was fixed to the wall inside the 3-mm tube and then a 0.5-mm hole was drilled 15 mm from the leading edge, i.e., at L/D=5. "Ears" were cemented under the tube at the aft part, ensuring that the probe tip would touch the wall even for curved surfaces. The probe was attached to the wall by means of tape over these "ears." The probe was located on a flat plate in the FFA  $0.4\times1$  m low-speed tunnel 1 m behind the plate leading edge. At the same streamwise station, a Preston tube was located at a lateral distance of 40 mm. For the purpose of calibration the freestream velocity was varied from 7 to 22 m/sec. The Preston tube readings were related to the true local skin friction by means of the expression:

$$\frac{\Delta p}{\tau} = 38.85 \cdot \log_{10} \frac{\Delta p D^2}{\rho \nu^2} - 111.92 \left[ \frac{\mathrm{d}p}{\mathrm{d}x} = 0 \right]$$
 (7)

Figure 1 shows  $C_{p^*}$ . It can be seen that as postulated it is not a function of  $U_\tau D/\nu$ . The rather large scatter is due to the fact that the magnitude of  $p_\star - p_s$  is small in physical quantities. An average value is  $C_{p^*} = -9.2$  for this probe.

A separate investigation of the function f(L/D) for zero pressure gradient flow was undertaken with a tube of diameter 5 mm for L/D ratios from 2 to 5. Although the tube was too large for total pressure measurements, the static pressures measured would reveal any important pressure gradients along the tube. Plotting the average  $C_p$  value for each of the eight L/D ratios yields a small variation from  $C_p$ ,  $\sim -10.5$  when L/D=2 to  $C_p$ ,  $\sim -9.2$  for L/D=5.

With reasonable accuracy it is therefore possible to use a calibration for the total head/static tube of form:

$$\frac{\Delta p_*}{\tau} = 38.85 \log_{10} \left[ \left( \frac{\Delta p_*}{\tau} - 4.6 \right) \left( \frac{U_\tau D^2}{v} \right] - 107.3 \quad (8a)$$

But  $\Delta p_*/\tau > > C_p \cdot /2$  for the values in question, and the simplified equation (8b) in the following is accurate within less than one percent as compared with (8a)

$$\frac{\Delta p_*}{\tau} = 38.85 \cdot \log_{10} \left( \frac{\Delta p_* D^2}{\rho v^2} \right) - 107.3 \left[ \frac{\mathrm{d}p}{\mathrm{d}x} = 0 \right]$$
 (8b)

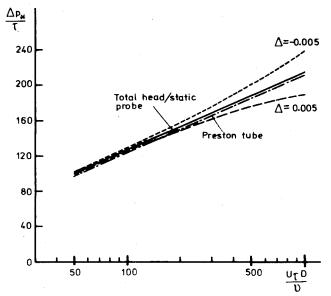


Fig. 2 Calibration curve for total head/static probe relative to Preston tube calibration. The pressure gradient effects are indicated as the additive term  $U_{\tau}D/\nu \cdot L/D \cdot \Delta$ , L/D = 5.  $\Delta = \nu/U_{\tau}^{3} \cdot 1/\rho \cdot \mathrm{d}\rho/\mathrm{d}x$ .

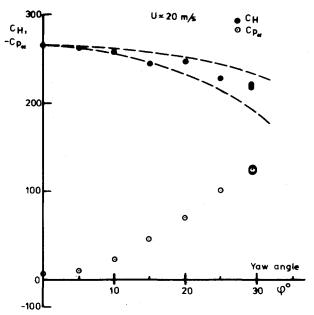


Fig. 3  $C_H$  and  $C_p$ , as functions of yaw angle  $\varphi$ , — Outer limits of data in Ref. 8. It should be noted that the level of  $C_H$  corresponds to the Preston tube reading.

This expression is based on  $C_p$ , = -9.2, a result obtained in zero pressure gradient flow. The addition of the pressure gradient term in Eq. (6) approximately accounts for moderate pressure gradient effects, under the assumption that the pressure gradient will not affect  $C_p$ , to first order.

The test range of this probe was  $U_{\tau}D/\nu = 70$  to 200, but the analysis indicates that the calibration is valid at least in the range 50 to 1000. Figure 2 illustrates this curve compared with the corresponding Preston tube curve. The effect of pressure gradient is also indicated.

One might fear that the location of the static pressure tap on top of the cylinder would cause the probe to be sensitive to yaw. A Preston tube in yaw tends to give lower output only due to change in the total pressure  $p_{tot}$ , while  $p_s$  may be regarded as constant.

However, for the new probe it turned out that the static pressure  $p_*$  changed rather slowly at small yaw angles and the response is in the opposite sense, as seen in Fig. 3.  $C_H = p_{tot}$ 

 $p_s/(\rho/2)U_7^2$ . In the figure the yaw variation of  $C_H$  according to Rajaratnam and Muralidhar 8 has been indicated.

#### **Conclusions**

A new method for measuring skin friction by means of a total head/static probe in a turbulent boundary layer has been described and a calibration has been given and discussed.

- 1) The probe measures the total head pressure at the tip and a static pressure at a location downstream of it (in the center plane). By accounting for the position error  $C_{p^*}$  both true static pressure and skin friction are determined.
- 2) The particular probe tested had the configuration D=3 mm and L/D=5, which is to be regarded a reasonable choice for most applications. Variation of the ratio L/D showed that at least down to L/D=2 there is no change in static pressure.
- 3) The sensitivity to misalignment is about twice that of a Preston tube and the total head/static probe responds in the opposite sense. It should be noted, though, that this statement does not consider the cross flow effects in three-dimensional boundary layers. If the variation in cross flow is small over the probe diameter one may nevertheless expect deviations comparable to the misalignment errors discussed previously.

# Acknowledgment

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# Boundary Conditions with Heat/Mass Transfer and Velocity Slip

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M OST physical problems of the boundary-layer type which are of general interest to the fluid mechanist

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